Spiking neural network methodology for modelling, classification and understanding of EEG spatio-temporal data measuring cognitive processes

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A B S T R A C T

The paper offers a new methodology for modelling, recognition and understanding of electroencephalography (EEG) spatio-temporal data measuring complex cognitive brain processes during mental tasks. The key element is that mental tasks are performed through complex spatio-temporal brain processes and they can be better understood only if we model properly the spatio-/spectro temporal data that measures these processes. The proposed methodology is based on a recently proposed novel spiking neural network architecture, called NeuCube as a general framework for spatio-temporal brain data modelling. The methodology is demonstrated on benchmark cognitive EEG data. The new approach leads to a faster data processing, improved accuracy of the EEG data classification and improved understanding of this data and the cognitive processes that generated it. The paper concluded that the new methodology is worth exploring further on other spatio-temporal data, measuring complex cognitive brain processes, aiming at using this method for the development of the next generation of brain–computer interfaces and systems for early diagnosis of degenerative brain disease, such as Alzheimer's Disease (AD), and for personalised neuro-rehabilitation systems.

1. Introduction

Modelling brain data, for the purpose of recognising and understanding of complex brain processes is a major problem in information sciences related to important applications, such as:

– Brain–computer interfaces (BCI) [1,44,54].
– Early diagnosis and prevention of degenerative brain diseases, such as Alzheimer's Disease (AD) [42,49,50,58].
– Personalised disease prognosis and neuro-rehabilitation [6,37].

Cognitive brain processes are characterised by complex spatio-/spectro-temporal brain data (STBD) that is difficult to process and understand in a computer model, unless we have a proper computational model that is relevant to the data. Many
brain data related applications from the above listed still use inappropriate techniques and require more efficient ones as
discussed below.

BCIs aim at decoding brain signals that represent cognitive processes to enable human–computer communication
[1–4,25,29,44,54]. BCIs still use traditional statistical and artificial intelligence (AI) methods for the recognition of complex
STBD. This often limits the functioning of the BCIs and leads to poor performance [40,54].

The increase in longevity of people’s lives in modern society brought also the urgency to face the problem of dealing with
the dramatic rise of neurological disorders and above all AD even in early onset [29,38,42,49,50,58]. This is becoming a major
public health problem that raises serious health questions and would require reorganisation of social care services. Cognitive
tests and memory screening are commonly used to stage and diagnose cognitive impairment, such as AD [57]. In fact, one
of the first symptoms that follow the onset of the pathology is cognitive decline and memory loss. Early detection is important,
as it allows for initial treatments. In this field, neuroinformatics research can play a pivotal role. A main contribution that can
be brought by information scientists is a much more efficient machine learning technique that can be used to evaluate the
cognitive ability of a person with a deeper understanding of the brain processes that cause it.

Personalised medicine is a current trend in health care with a huge potential in many health related areas, including BCI
and neuro-rehabilitation robotics [6,70]. Still new information science methods are needed for the efficient implementation
of this concept as it has already been demonstrated in [37].

Electroencephalogram (EEG) data is the most commonly collected data for the study of brain processes [1,9,10,13,25,29,55]
related to both BCI and brain diseases. However, this data is the product of complex spatio-temporal brain signal pathways
that are time dependent events related to the brain structure and its functioning [29,64,67,68]. In principle, EEG data is
spatio-/spectro temporal brain data (STBD) and it has to be processed as such in order to be properly understood. This has been
done with only partial success so far which raises the need for the development of new information science methods as pointed
in [19,29,34].

In this paper we propose a novel methodology for the classification of EEG STBD recorded during different cognitive activ-
ities (Section 2). The methodology is based on the brain inspired spiking neural networks (SNN) and more specifically – on a
recently proposed SNN architecture NeuCube [32,34]. In Section 3 we demonstrate the methodology on a benchmark data
recorded during cognitive tasks [9]. Section 4 compares the proposed method and the obtained results on the benchmark
data with traditional approaches, with the conclusion that the new method not only results in a significantly better accuracy
of classification, but also in a better interpretation of the model and a better understanding of the data and the cognitive
processes that generated it. Accurate classification results can be interpreted as an indicator for the detection of memory
related cognitive problems. Section 4 highlights future directions for research based on the proposed methodology.

2. A spiking neural network methodology for modelling of EEG spatio-temporal data of brain cognitive activities

2.1. Brain-like SNN for modelling spatio-temporal data

There is a vast amount of information about structural and functional characteristics of the human brain accumulated so
far, including: synaptic processes [29]; information encoding [65]; connectivity [27]; structural and functional atlases [
43,46,47,67]; cortical projections [41]; genetics and proteomics [5,21]; neuro-genetic atlases [20,28]; brain disorders and
conditions [57]. All this can be valuable information when processing cognitive data if our computational models can rep-
resent it.

From information science point of view, the brain represents and processes information at a ‘low level’ in the form of
many trains of temporal electrical potentials that can be considered binary events (spikes) and are transferred between
neurons through synaptic connections. Through learning from data the synaptic connections are modified to reflect more
precisely the timing of the data from the sensory inputs. And this is one of the principles of SNN, considered the third
generation of brain-inspired neural network techniques [16–19]. SNN methods have been developed for: learning from data
[17,22,26,45,62,66]; system design and implementation [17,19,59]; encoding continuous input data into spike trains, such as
the silicon retina [8] and the silicon cochlea sensory devices; neurogenetic computation [5,31,35,38]; high performance and
neuromorphic engineering systems [14,15,24,53]. Promising features of SNN are: compact representation of space and time;
fast information processing; time-based and frequency-based information representation. Methods of SNN for spatio-
temporal pattern recognition have been already developed [11,23,26,33], including: evolving SNN (eSNN) classifiers
[30,36,48,59,71]; pilot applications for moving object recognition [36,59]; pilot applications for simple EEG data classifica-
tion [6,55]; SNN reservoir computing and liquid state machines [59,61,69]; finite automata modelling [52,53].

SNN methods and techniques provide a solid background for the development of new, brain-like information methods
and systems for STBD. One of them, called NeuCube, is described below and used in the next sections for the proposed in
this paper methodology for cognitive EEG data modelling, classification and understanding.

2.2. The NeuCube SNN architecture for modelling STBD

NeuCube is a SNN architecture for STBD, initially proposed in [32] and then further developed in [6,34,37,61]. A block
diagram of the NeuCube model is depicted in Fig. 1. The NeuCube architecture consists of the following modules: input
information encoding module; 3D SNN cube module (SNNc); output (classification) module; gene regulatory network (GRN) module (optional); optimisation module (optional).

The input module transforms input data into trains of spikes that are entered then into the main module – the 3D SNNc. The SNNc is structured to spatially map brain areas for which STBD is available. That may include known structural or/and functional connections between different areas of the brain represented in the data. Setting up a proper initial structural connectivity in a model, in our case the SNNc, is important in order to learn properly spatio-temporal data and capture functional connectivity. More specific structural connectivity data can be obtained using for example diffusion tensor imaging (DTI) method.

The initial structure of the SNNc can be preliminary defined based on the available brain data and the problem or/and generated as a small-world connectivity [34]. The structure is also evolving through the creation of new neurons and new connections based on the ECOS principles used in eSNN classifiers [30,36,48,55]. If new data do not sufficiently activate existing neurons in the output classifier, new neurons are created and allocated to match the data along with their new connections.

Learning in the NeuCube is performed in two stages:

- Unsupervised learning, where STBD in the forms of spike trains, is entered into corresponding areas of the SNNc. Unsupervised learning is performed to modify the initially set connection weights. The SNNc will learn to activate same groups of spiking neurons when similar input stimuli are presented, also known as a polychronization effect [26].
- Supervised learning of the spiking neurons in the output classification module, where the same STBD used for the unsupervised training, is now propagated again through the trained SNNc and output neurons are generated (evolved) and trained to classify the spatio-temporal spiking pattern of the SNNc into pre-defined classes (or output spike sequences). As a special case, all neurons from the SNNc are connected to every output neuron. Feedback connections from output neurons to neurons in the SNNc can be created for reinforcement learning. Different SNN methods can be used to learn and classify spiking patterns from the SNNc, including: eSNN [30,71]; dynamic evolving SNN (deSNN [36]); spike pattern association neuron (SPAN [48]); other SNN classifiers [59]. All these options are implemented in some preliminary NeuCube implementations [6,34,61]).

Memory in a NeuCube-based model is represented as:

- Short-term memory, represented as changes of the post-synaptic potentials (PSP) and temporary changes of synaptic efficacy.
- Long-term memory, represented as a stable establishment of synaptic efficacy – long-term potentiation (LTP) and long-term depression (LTD).
- Genetic memory, represented as a genetic code.
NeuCube is a new type of computational architecture, which allows the creation of different models for STBD based on the following information processing principles as listed in [34]:

(1) The model has a spatial structure that maps approximately the spatially located areas of the brain where STBD is collected.
(2) The same information paradigm – spiking information processing that ultimately generates STBD at a low level of brain information processing, is used in the model to represent and to process this STBD.
(3) Brain-like learning rules are used in the model to learn STBD, mapped into designated spatial areas of the model.
(4) A model is evolving in terms of new STBD patterns being learnt, recognised and added incrementally, which is also a principle of brain cognitive development.
(5) A model always retains a spatio-temporal memory that can be mined and interpreted for a better understanding of the cognitive processes.
(6) A visualisation of the model evolution during learning can be used as a bio-feedback.

All the above principles make NeuCube a suitable SNN architecture to learn and reveal complex spatio-temporal patterns ‘hidden’ in STBD and that is why it has been chosen for the development of the new methodology to model cognitive EEG STBD as outlined in the next sub-section.

2.3. A NeuCube-based methodology for modelling EEG spatio-temporal data of brain cognitive activities

The proposed here methodology for modelling, classification and understanding of EEG cognitive data consists of the following procedures:

(1) Collecting EEG STBD representing relevant to the study brain cognitive processes.
(2) Encoding the EEG data into spike sequences.
(3) Mapping the spike sequences into a specially designed SNNc that reflects on the number of input variables (channels) and the data available.
(4) Training the SNNc on the spike STBD using unsupervised learning method, e.g. STDP [62].
(5) Training of an output classifier in a supervised mode.
(6) Validation of the classification results.
(7) Repeating steps (2) to (6) for different parameter values in order to optimise the classification performance. Record the best performing model.
(8) Visualise the trained SNNr and analyse its connectivity and spiking activity for a better understanding of the data and the brain processes that generated it.

For mapping the spatial locations of the EEG channels into the SNNc, we use the Talairach template [41,43,63]. The neurons in the SNNr are located following the same \((x, y, z)\) coordinates of the Talairach template and the EEG channels are mapped according to the standard mapping suggested in [41]. The spike sequences that represent data from EEG channels (after transformation of continuous value signals into spike trains) are entered into the correspondingly located neurons in the SNNc during the unsupervised training procedure.

The above methodology is applied step-wise on a case study problem of cognitive EEG data modelling in the next section. Figs. 2–4 illustrate the procedures above.

3. A case study on EEG cognitive data modelling

3.1. Data description

The data used for this study was recorded in an earlier experiment [9,39,40] and further studied in [1,3,51]. This data was collected from the cortex of seven healthy subjects (between 20 and 48 years old; six men and one woman; all right-handed except for one subject) following five different scenarios, one resting task and four cognitive tests. A brain computer interface device was used to collect EEG data. The designed mental task scenarios consisted of: a “resting” task – a subject is relaxing, avoiding thoughts as much as possible (class 1); a “letter composition” task (class 2) – a subject is tasked with imagining writing a letter to someone without verbally expressing it; a “multiplication” task (class 3) – a subject is performing a non-simple two digit mental multiplication; a “counting” task (class 4) – a subject is visualising a blackboard on top of which numbers were sequentially being written; a “rotation” task (class 5) – a subject is mentally rotating a 3D geometric figure. Each recording session was carried out using six electrodes: C3, C4, P3, P4, O1, O2. Data was recorded for 10 s at 250 Hz, resulting in 2500 data points collected per session. Every task was repeated five times during a daily session. Some of the subject data was recorded on a one-day session, while other subjects repeated the five trial tasks for a second or third day session. The data of subject 4 was excluded from the experiment, as according to a previous study [51], the signal was repeatedly saturated or invalidated in several trials.
Fig. 2. A graphical representation of the different steps from the proposed methodology applied on the case study problem.
For our study, we resized each session dataset into two samples of 5 s each, 1250 data points per channel on every sample. We assumed that half of the information can be considered enough to represent the mental activity measured. Thus, for each of the 5 classes we had 10 samples of 1250 data points/C2 EEG channels, in total, we obtained 50 samples per subject and per session.

3.2. Model design and implementation

We applied the methodology from Section 2 on the above experiment as shown graphically in Fig. 2. For the model implementation we have used a software simulator of NeuCube written in MATLAB (see [6,34] for more details).

EEG STBD was classified per subject and per session using a NeuCube-based model that had a SNNc of 1471 spiking neurons using the Talairach EEG electrode mapping template [41]. One of the advantages of the NeuCube framework is that in many cases there is no need of pre-processing (such as normalisation of the data, scaling, and smoothing). The raw data is fed into the model as time series and transformed into spike trains using address event representation method (AER [8]), and then the transformed spike trains are mapped into the SNNr for unsupervised training. The AER method is suitable for EEG cognitive STBD, as this algorithm identifies just differences in consecutive EEG values.

The input spike sequences are presented to the SNNc, which was implemented using leaky integrate and fire (LIF) neurons, which is less computationally expensive [17,24,61]. The SNNc was trained using the spike timing dependant plasticity (STDP [62]) learning rule. The STDP learning rule allows the spiking neurons of the SNNc to learn consecutive temporal associations from the EEG data within-, and across EEG channels, and therefore forming new connections in the architecture that can be analysed and interpreted. This makes the NeuCube architecture useful for learning spatio-temporal patterns from EEG data, forming associative type of memory that can be further explored [34].

Although the SNNc can be evolving in size, for this research we have explored the classification ability of the NeuCube architecture of 1471 spiking neurons, each representing the centre coordinates of a one cubic centimetre area from the 3D Talairach Atlas [43,63].
The 3D architecture of the SNNc is initialised as “small-world” (SW) connected networks. The SW connectivity principle is fundamental for the initialization, the learning processes of this model and for the process of capturing relevant patterns from the data. The encoded into spike sequences data from the six EEG recording device channels (C3, C4, P3, P4, O3 and O4) is entered as time series into spatially allocated neurons following the Talairach coordinates as suggested in [41] (Fig. 3). Fig. 3 also shows different areas of the SNNc that spatially represent regions of the brain according to the Talairach’s template: frontal lobe, temporal lobe, parietal lobe, occipital lobe, posterior lobe, sub-lobar region, limbic lobe, anterior lobe, coloured in different colours.

As an output classifier we used the dynamic evolving SNN (deSNN [36]) algorithm to classify the EEG STBD into the 5 brain cognitive states (classes). This classification method combines the rank-order learning rule [66] and the STDP [62] temporal learning for each output neuron to learn a spatio-temporal pattern using only one pass of data propagation. The classification results were evaluated using both repeated random sub-sampling validation (RRSSV) and leave-one-out cross-validation (LOOCV).

The last picture of the diagram in Fig. 2 represents another key advantage that NeuCube offers: the possibility of model interpretation and knowledge extraction for the purpose of a better understanding of the data and the cognitive brain processes. The state of the SNNc after training can be visualised in different ways and analysed. It can be observed that new connections are formed between neurons that can be further interpreted in the context of different cognitive tasks.

It is important to highlight that the NeuCube model is a stochastic model (i.e. initial connection between the neurons of the reservoir are randomly generated) and therefore the model is sensitive to parameter settings. Some of the major parameters that highly influence the model are:

- The AER threshold of the encoding spike trains – a bi-directional threshold, which is applied to the signal’s gradient according to the time. When input EEG data is loaded, it is transformed into spike strains. The spike rates depend on the AER threshold, which can be determined either as a particular value for every input variable or as a global threshold to be applied to all of them.
- Connectivity between neurons of the network. Depending on a SW connectivity parameter, each neuron of the SNNr is initially connected to its neighbouring neurons within this parameter as distance. We have used a value of 0.15.
- The threshold of firing, the refractory time and the potential leak rate of the LIF neurons. When a LIF neuron of the reservoir receives a spike, its PSP increases gradually with every input spike according to the time, until it reaches an established threshold of firing. Then, an output spike is emitted and the membrane potential is reset to an initial state (refractory time). Between spikes, the membrane potential leaks. In our experiments the three parameters are set to 0.5, 6 and 0.02 respectively.
- The STDP rate parameter. According to the STDP learning rule (see [62] for more details), the firing activity of two connected neurons causes their connection weight to increase or decrease depending on the order of firing, so that the connection weight will reflect on the temporal relationship between the activities of these neurons. The experiment here uses a value 0.01.
- The number of times that the NeuCube is trained in an unsupervised mode. This is set by default as 2, as higher values may cause over training of the SNNc.
- The variables mod and drift of the deSNN classifier. According to [36], every training sample is associated to an output neuron, which is connected to each and every other neuron of the cube. The initial connection weights of these output neurons are set as zero. New connection weights are formed according to the rank-order (RO) learning rule. This are calculated depending on a modulation factor (the variable mod) of the order of the incoming spikes. The new connection weights will then increase or decrease according to the number of spikes that follow the first one (the drift value). We have used for these parameters values of 0.4 and 0.25 correspondingly.

A crucial step in obtaining good results from the NeuCube model is the optimisation of these parameter values. Parameter optimisation can be achieved via grid search method, genetic algorithm, or quantum-inspired evolutionary algorithm [7,56]. In this study we applied a grid search using 50% of the entire time series for training and the other 50% for testing, both randomly selected. We assessed the classification accuracy of 20 model configurations for each subject and for each session using different AER values and used the AER that resulted in the best accuracy (Table 1). The other parameter values were set as explained above. These optimized parameters were used to evaluate the classification output in all experiments including LOOCV method.

3.3. Experimental results

In this study, we measured the classification accuracy of the NeuCube-based models (Table 1) and the average time for execution (Table 2). Table 1 summarises classification results per subject and per session. Results are expressed as a percentage of accurately classified samples per class type and over all classes. The results are obtained using randomly selected 50/50% train/test data and LOOCV method.

As the data set was of a small size, it is not appropriate to draw any scientific conclusions about the mental tasks performance by different subjects, and that was not the goal of this paper. We rather conclude that it is feasible to consider the NeuCube-based method for further analysis and further experimental data modelling to become a widely used method.
for EEG data analysis related to mental tasks and cognitive processes across applications. The results from this experiment still confirmed some expected phenomena:

- Subjects perform differently for different complex mental tasks (classes).
- Data for class 1 (relax) was the best classified across all subjects.
- The accuracy of classification increased with some manual parameter tuning showing that this is not the full potential of the NeuCube-based model and it still needs to be further optimised.
- Even dealing with very complex mental tasks, the classification accuracy was better when compared to previously used classification models.

The above comparison was confirmed as the results from the NeuCube based method were compared with the results obtained in previous experiments carried out the same data set [51]. With the proposed NuCube-based method we obtained higher classification accuracy on the data per session, per subject and overall (see Table 3) when compared with other methods such as support vector machines (SVM) and extreme learning machines (ELM) (tested in a leave-one-out cross validation mode). When the SVM and the ELM methods were applied, the EEG data was first pre-processed (smoothed) and then – 'compressed' into smaller number of input vectors, rather than treated as spatio-temporal stream data as it is in the NeuCube-model case.

Table 2
CPU time in seconds for the main NeuCube’s algorithms (data encoding, neucube initialization, unsupervised training of the cube and supervised training-testing of the classifier).

<table>
<thead>
<tr>
<th>CPU time (s)</th>
<th>Data encoding</th>
<th>NeuCube initialization</th>
<th>Unsupervised training</th>
<th>Classifier ST and T</th>
</tr>
</thead>
<tbody>
<tr>
<td>(whole data)</td>
<td>48.46 (per sample per iteration)</td>
<td>20 (per sample)</td>
<td>5854.51</td>
<td></td>
</tr>
</tbody>
</table>

The reported in Table 2 average CPU time has been measured under the following settings: a software simulator of NeuCube written in MATLAB R2012b\(^6\); a 64 bit Windows 7 machine, with processor Intel\(^\circ\) core\(^\text{TM}\) i5-2400 CPU 3.10 GHz; 8 GB of memory (RAM).

In addition to the above, the NeuCube-based model has several other important advantages:

- It requires only one iteration data propagation for learning, while the classical methods of SVM and ELM require hundreds of iterations.
- The NeuCube-based model is adaptable to new data and new classes, while the other models are fixed and difficult to adapt on new data.
- The NeuCube-based model allows for a good interpretation of the data as discussed next.

3.4. Model interpretation for a better understanding of data

NeuCube constitutes a biologically inspired three-dimensional environment of SNN for on-line learning and recognition of spatio-temporal brain data. It takes into account data features, offering a better understanding of the information and the phenomena of study. This is illustrated in Fig. 4 which was obtained after the SNNc was trained with one of the data sets. From Fig. 4 we can notice that new connections are formed around the input neurons of the SNNr which were allocated so that they spatially map the spatial location of the EEG electrodes. Studying the picture, we could also deduce some additional...
information, e.g. subjects where using actively their visual cortex (occipital lobes). Effectively, the subjects were performing each scenario with open eyes. We can also observe from the picture a high activity on the parietal lobe (integration of visual and other information) during the cognitive task related to this data.

The NeuCube model can be further trained incrementally on new data, including new classes, due to the capacity of the SNNr to accommodate data in one pass learning and to the evolvability of the output classifier. The latter will generate a new output neuron for every new input pattern learned and will train it in one pass learning mode [30,34,36]. This ability of the NeuCube models will allow to trace the development/decline of cognitive processes over time and to extract new information and knowledge about them.

4. Conclusion and future directions

The aim of this research has been to develop a methodology and a framework for modelling and interpretation of EEG data that measures brain activities during cognitive tasks. This is important for the creation of new types of BCI and also for early detection of cognitive decline to be used by clinicians in everyday diagnosis. For this purpose, we selected as a benchmark data EEG STBD on complex cognitive tasks [9,39,40]. In this study we proposed a methodology based on the novel SNN architecture called NeuCube [34], for classifying spatio-temporal EEG data collected while subjects were performing 4 types of cognitive tasks and a relax mode.

NeuCube offers several advantages when compared to traditional information methods:

- Fast learning of STBD (only one pass data propagation).
- Higher accuracy of classification.
- Ability to adapt to new data through incremental learning (an evolving SNN is used as an output classifier [30,36,48]), that includes learning of new input patterns from data and new classes.
- Interpretation of the model for a better understanding of the EEG STBD and the processes that generated it.

Future research directions include:

- Experimentation of NeuCube-based models on other EEG data, e.g. [12].
- Parameter optimisation using quantum inspired evolutionary algorithms [7,56,60].
- Adding genetic information in terms of GRN [5,20,21,29,38] to the model to help study the impact of genes on cognitive abilities, e.g. how much gene expression levels of neuro-receptors affect certain cognitive tasks.
- Testing the proposed method on new types of BCI, including neuro-rehabilitation [6,70].
- Testing the proposed method in clinical environment for early diagnosis of cognitive decline.
- Extending the proposed method for predictive modelling and personalised prognosis [37].
- Improved visualisation of the SNNr and the classifier during the training and recall procedures for an improved understanding of the data and the brain processes.
- Implementation of the proposed method on neuromorphic hardware to explore its potential for a highly parallel computation [14,15,24,53,61].

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