CHANNEL EQUALIZATION USING ADAPTIVE FILTERING WITH AVERAGING

Georgi Iliev and Nikola Kasabov
Department of Information Science, University of Otago
P.O. Box 56, Dunedin, New Zealand
giliev@infoscience.otago.ac.nz, nkasabov@otago.ac.nz

***Abstract* – The recent digital transmission systems impose the application of channel equalizers with short training time and high tracking rate. These requirements turn our attention to adaptive algorithms, which converge rapidly. From this point of view the best choice is the recursive least squares (RLS) equalizer. Unfortunately this equalizer has high computational complexity and stability problems. Therefore it is worth investigating a good alternative of the classic RLS equalizer. In this contribution we present such a channel equalizer based on adaptive filtering with averaging (AFA). The main advantages of AFA equalizer could be summarized as follows. It has high convergence rate comparable to that of the RLS equalizer and at the same time low computational complexity and possible robustness in fixed-point implementations.**

I. INTRODUCTION

One of the most important advantages of the digital transmission systems for voice, data and video communications is their higher reliability in noise environment in comparison with that of their analog counterparts. Unfortunately most often the digital transmission of information is accompanied with a phenomenon known as intersymbol interference (ISI). Briefly this means that the transmitted pulses are smeared out so that pulses that correspond to different symbols are not separable. Depending on the transmission media the main causes for ISI are:

- cable lines – the fact that they are band limited;
- cellular communications – multipath propagation.

Obviously for a reliable digital transmission system it is crucial to reduce the effects of ISI and it is where the adaptive equalizers come on the scene [1], [2]. Two of the most intensively developing areas of digital transmission, namely digital subscriber lines (DSL, HDSL, ADSL and VDSL) and cellular communications (GSM and IS-54) are strongly dependent on the realization of reliable channel equalizers [3], [4], [5]. One of the possible solutions is the implementation of equalizer based on filter with finite impulse response (FIR) [6], [7] employing the well known least mean squares (LMS) algorithm for adjusting its coefficients [8], [9]. An enhancement of this equalizer is the so-called decision feedback equalizer, which is a combination of two adaptive filters. These two variants are most often encountered in practice [10] and they work as a rule in two modes:

- training – the adaptive equalizer identifies the channel characteristics and works with known (training) sequence;
- tracking – the equalizer operates in so-called decision direction fashion.

Recently blind equalizers have received great research interest [11], [12] since they do not require training sequence and extra bandwidth. The main weaknesses of these approaches are their high computational complexity and slow adaptation [12].

Namely the requirement for fast adaptation turns out the most important factor in applications where the changes in channel characteristics could be quite rapid. This is the basic stimulus for the utilization of adaptive algorithms, which have a good rate of adaptation, of course, at the price of more computations. An example is Kalman/Godard algorithm [13] also known as recursive least squares algorithm (RLS) [14]. Kalman filter maintains the requirement for fast convergence but it has two disadvantages:

- high computational complexity;
- stability issues [15], which are mainly due to the method for updating the Kalman gain vector.

To summarize the requirement for fast adaptation rate, which is vital especially in applications as cellular communications where the changes in channel characteristics may be quite rapid, forces the investigation of adaptive algorithms that could be a good alternative of the classic Kalman/Godard algorithm. They are expected to avoid the problems related to high computational complexity and low stability in fixed-point implementations. The need of such algorithms is the main motivation for the investigations conducted in this contribution. Here we present the results of the realization of an adaptive equalizer using adaptive filtering with averaging. The results show that this channel equalizer could be a good alternative of Kalman equalizer maintaining its fast adaptation rate and at the same time eluding its basic weaknesses.

II. CHANNEL EQUALIZATION

As mentioned in the introduction the intersymbol interference imposes the main obstacles to achieving increased digital transmission rates with the required accuracy. Traditionally, ISI problem is resolved by channel equalization in which the aim is to construct an equalizer such that the impulse response of the channel/equalizer combination is as close to z^{-d} as possible, where d is a delay. Frequently the channel parameters are not known in advance and moreover they may vary with time, in some applications significantly. Hence, it is necessary to use the adaptive equalizers, which provide the means of tracking the channel characteristics.

In Fig. 1 a digital transmission system using channel equalization is depicted. Here, as most often in practice, the analog signal is sampled at Nyquist rate and then the different samples are coded with binary sequences taking the two possible values –1 and 1. In Fig. 1 s(n) denotes the transmitted signal where the sampling period T=1. Actually s(n) is a random sequence of –1s and 1s. Here the channel is dispersive and can be modeled by a FIR filter. Thus the channel output is written as

$$x(n) = \sum_{i=0}^{N} a_i s(n-i) \qquad (1)$$

where N is filter order and a_i, $0 \le i \le N$, are filter coefficients.

In addition

$$y(n) = x(n) + n(n) \qquad (2)$$

is the adaptive equalizer input, where n(n) is the inevitably present additive noise.

Obviously the problem to be considered is that of using the information represented in the observed equalizer inputs y(n) to produce an estimate of transmitted symbols in the sequence s(n). There is a delay through the adaptive equalizer, which means that the estimate is delayed by d symbols. In Fig.1 this is denoted as d(n-d) that is an estimate of s(n-d). In the scheme shown in Fig. 1 a decision device applies a set of thresholds to recover the original data symbols selecting the symbol which is closest to the estimate d(n-d). For example, if the transmitted signal takes values ±1, the decision device would simply be

$$\bar{d}(n-d) = \mathrm{sng}\ [d(n-d)] \qquad (3)$$

where sng [ ] is the sign function. Here the object is that $\bar{d}(n-d)$ = s(n-d).

In such a digital transmission system a transversal filter can be used as an adaptive equalizer. This arrangement is shown in Fig. 2 as the equalizer forms the linear combination of input samples as follows

$$d(n\text{-}d) = \sum_{j=-d}^{d} w_j(n) y(n-d-j) \qquad (4)$$

where w_j, $-d \le j \le d$, are the filter coefficients. The object of the adaptive algorithm is to adjust the filter coefficients in such a manner that d(n-d) ≈ s(n-d). A criterion that can be applied to adapt the coefficients is the minimization of the output mean square error $E[e^2(n\text{-}d)]$ where

$$e(n\text{-}d) = d(n\text{-}d) – s(n\text{-}d). \qquad (5)$$

In (5) the estimation error is determined as the difference between the estimate and the original signal, which implies that the transmitted sequence is known in advance. In this case the equalizer operates in so-called training mode.

The second possible fashion of working is decision direction mode where

$$e(n\text{-}d) = d(n\text{-}d) - \bar{d}(n-d)\,. \qquad (6)$$

Notice that here the estimation error is defined as the difference between the estimate and the detected data symbols at the decision device output. This manner of adaptation is acceptable for tracking a slowly varying channel, but can not be used for initial channel identification. In this case the approach is to utilize a deterministic data sequence and a training period. The time it takes for training of the equalizer is strongly dependent on the choice of adaptive algorithm. The main concern is to reduce the training time as much as possible and this suggests the usage of adaptive algorithms, which converge rapidly.

Two of the most widely spread algorithms in adaptive equalization practice are LMS and RLS algorithms. For the adaptive channel equalizer depicted in Fig. 2 the needed steps for these algorithms are presented in Table 1 and Table 2. Comparing the two algorithms, it is clear that the strongest point of LMS algorithm is its low computational complexity. Unfortunately this algorithm has low adaptation rate. In contrast, RLS algorithm provides a tracking rate sufficient for a fast-fading channel, but the price to be paid is an order-of-magnitude increase in complexity. Moreover RLS algorithm is known to have stability issues [16] due to the covariance update formula P(n) (see Table 2). This fact creates a lot of problems especially in fixed-point implementations. All mentioned above turns our attention to search for adaptive algorithms, which have fast convergence that means short training time and good tracking rate and at the same time are free of the basic RLS' shortcomings, namely high complexity and stability problems. In the next section we present such a candidate algorithm to be used in channel equalization.

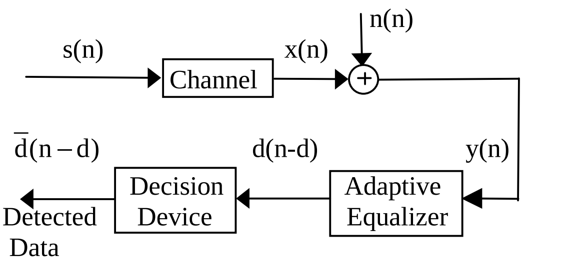

Fig. 1. Digital transmission system using channel equalization.

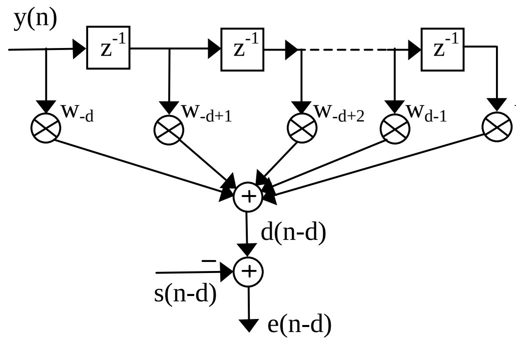

Fig. 2. Transversal channel equalizer.

Table 1. LMS equalizer.

Signal estimation:

$$\text{d(n-d)} = \sum_{j=-d}^{d} w_j(n)y(n-d-j)$$

Signal detection:

$$\bar{d}(n-d) = \text{sng}\ \ [d(n-d)]$$

Estimation error:

$$e(n-d) = d(n-d) - s(n-d) \text{ - training}$$

$$e(n-d) = d(n-d) - \bar{d}(n-d) \text{ - tracking}$$

Coefficients update:

$$w_j(n+1) = w_j(n) - \mu e(n-d)y(n-d-j)$$

$$\text{for } -d \le j \le d$$

Table 2. RLS equalizer.

Signal estimation:

$$\text{d(n-d)} = \sum_{j=-d}^{d} w_j(n)y(n-d-j)$$

Signal detection:

$$\bar{d}(n-d) = \text{sng}\ \ [d(n-d)]$$

Estimation error:

$$e(n-d) = d(n-d) - s(n-d) \text{ - training}$$

$$e(n-d) = d(n-d) - \bar{d}(n-d) \text{ - tracking}$$

Covariance update:

$$P(n) = P(n-1) - \frac{P(n-1)Y(n)Y^T(n)P(n-1)}{1/\delta + Y^T(n)P(n-1)Y(n)}$$

Gain update:

$$K(n) = \frac{P(n-1)Y(n)}{1/\delta + Y^T(n)P(n-1)Y(n)}$$

Coefficients update:

$$w_j(n) = w_j(n-1) - k_j(n)e(n-d)$$

$$\text{for } -d \le j \le d$$

$$0<\delta<1 \quad P(0) = \alpha I \quad \alpha \text{ large}$$

III. A CHANNEL EQUALIZER USING ADAPTIVE FILTERING WITH AVERAGING

In this section the application of an algorithm base on adaptive filtering with averaging in channel equalization is investigated. For simplicity the channel equalizer implemented by this adaptive algorithm is denoted as AFA (adaptive filtering with averaging) equalizer all through the rest of this paper. The main motivation for the utilization of such an algorithm are the results reported in [17] that show the good convergence properties of this kind of algorithms, which are vital for the requirements stated in the previous section.

Recalling the problem outlined in section II and using the same notations, we can re-define the problem in the following manner. To recursively update the vector of the filter coefficients as depicted in Fig 2, so that the estimation error given by (5) and (6) is minimized. A standard algorithm is of the form:

$$W(n+1) = W(n) - a(n)Y(n)e(n) \tag{7}$$

where

$W(n) = [w_{-d}(n), w_{-d+1}(n),\ldots, w_{d-1}(n), w_d(n)]^T$ is the coefficients vector,

$Y(n) = [y(n), y(n-1),\ldots,y(n-d),\ldots,y(n-2d)]^T$ is the input vector and a(n) is a sequence of positive scalars as $a(n) \to 0$ for $n \to \infty$.

In (7) the estimation error can be written as

$$e(n) = Y^T(n)W(n) - s(n). \tag{8}$$

The equation (7) could be transformed in two steps. First, we take the averages of W and in this way come to the following algorithm

$$W(n+1) = W(n) - \frac{1}{n^\gamma} Y(n)e(n) \tag{9}$$

$$\overline{W}(n) = \frac{1}{n}\sum_{k=1}^{n} W(k)$$

$$1/2<\gamma<1$$

The investigations presented in [17] show that such an algorithm could be unstable in the initial period. In order to improve the stability we undergo the second step, namely to average not only trough W but also through the product of Y and e. This leads us to the algorithm:

$$\overline{W}(n) = \frac{1}{n}\sum_{k=1}^{n} W(k)$$

$$W(n+1) = \overline{W}(n) - \frac{1}{n^\gamma}\sum_{i=1}^{n} Y(i)e(i) \tag{10}$$

$$1/2<\gamma<1$$

We term this algorithm AFA algorithm and the needed steps for the implementation of an AFA equalizer are presented in Table 3.

Going through Table 3 it could be concluded that first, the averaging here does not create additional burden since the terms $\overline{w}_j(n)$ and $\overline{ye_j}(n)$ can be recursively computed from their past values. Second, the algorithm does not use the covariance matrix, so there is no need of covariance estimate. This implies low computational complexity and escape from stability issues related to P(n).

Table 3. AFA equalizer.

Signal estimation:

$$d(n\text{-}d) = \sum_{j=-d}^{d} w_j(n)y(n-d-j)$$

Signal detection:

$$\overline{d}(n-d) = \text{sng}\ [d(n-d)]$$

Estimation error:

$$e(n\text{-}d) = d(n\text{-}d) - s(n\text{-}d) \text{ - training}$$

$$e(n\text{-}d) = d(n\text{-}d) - \overline{d}(n-d) \text{ - tracking}$$

Coefficients update:

$$\overline{w}_j(n) = \frac{1}{n}\sum_{k=1}^{n} w_j(k)$$

$$\overline{y\,e_j}(n) = \sum_{i=1}^{n} y(i-d-j)e(i-d)$$

$$w_j(n+1) = \overline{w}_j(n) - \frac{1}{n^{\gamma}}\overline{y\,e_j}(n)$$

for $-d \le j \le d$ and $1/2 < \gamma < 1$

IV. EXPERIMENTAL RESULTS

In this section we evaluate the performance of the proposed AFA equalizer. Since the main parameter of concern is the rate of adaptation, this parameter is tested and a comparison with the LMS and RLS equalizers is implemented. In the conducted simulations the transmitted signal s(n) is a random sequence of –1s and 1s, SNR=20 dB, the channel is modeled with a FIR filter of second order and the equalizer is realized as a FIR adaptive filter of 6^{th} order. The LMS, RLS and AFA equalizers are implemented according to the steps presented in Tables 1-3 as for the LMS algorithm - μ= 0.02, for the RLS algorithm - δ= 0.98 and for the AFA algorithm - γ = 0.5.

Under these conditions the trajectories of the equalizer coefficients are shown in Figures 3-5. Fig. 6 provides the results for MSE obtained over 100 independent trials. Obviously the AFA equalizer outperforms the LMS equalizer and its convergence rate is comparable to that of the RLS equalizer.

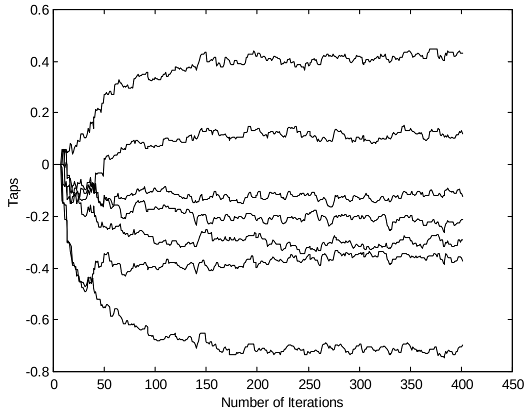

Fig. 3. Trajectories of equalizer taps – LMS.

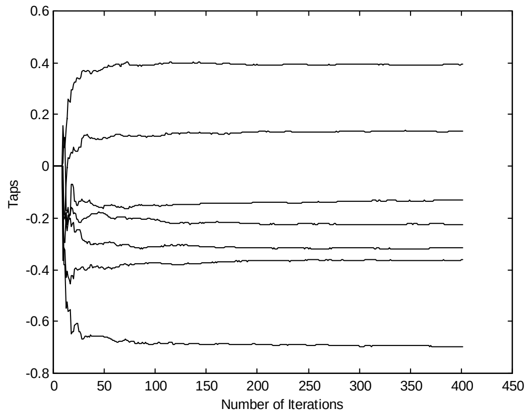

Fig. 4. Trajectories of equalizer taps – RLS.

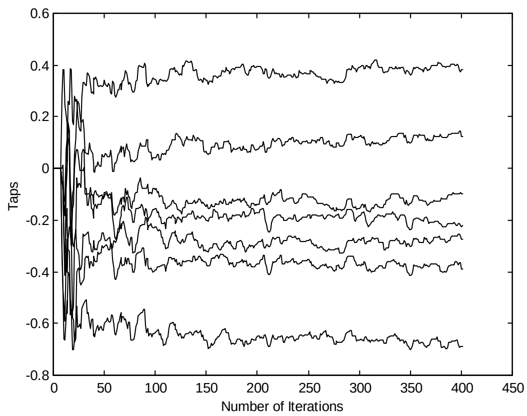

Fig. 5. Trajectories of equalizer taps – AFA.

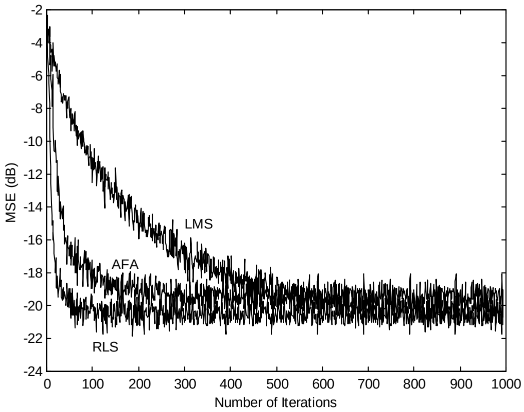

Fig. 6. Learning curves of LMS, AFA and RSL equalizers.

V. CONCLUSIONS

The basic goal of this paper is to investigate the application of an algorithm based on adaptive filtering with averaging in channel equalization. Here the main concern is to achieve a high convergence rate in order to meet the requirements for short training time and good tracking properties. In this light the obtained results show that the AFA equalizer is very promising. Its main advantages could be summarized as follows:

- high adaptation rate, comparable to that of the RLS equalizer;
- low computational complexity and possible robustness in fixed-point implementations.

A realization based on DSP to test the AFA equalizer performance in real environment is subject of future investigations.

A demonstration program is available on the WWW from: http://divcom.otago.ac.nz/infosci/KEL/CBIIS.html.

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